DOCUMENT RESUME

ED 028 948 24 SE 006 621

By-Davis, Robert B.

The Madison Project-A Brief Introduction to Materials and Activities.

Syracuse Univ., N.Y.; Webster Coll., Websters Grove, Mo.

Spons Agency-National Science Foundation, Washington, D.C.; Office of Education (DHEW), Washington, D.C. Bureau of Research.

Bureau No-BR-5-1172

Pub Date 65

Contract-OEC-6-10-183

Note-33p.

EDRS Price MF-$0.25 HC-$1.75

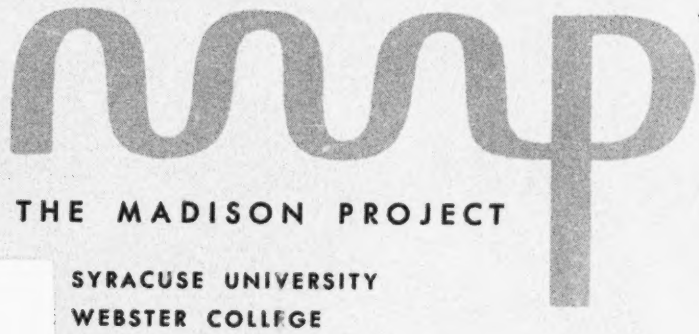
Descriptors-*Curriculum, Curriculum Development, Discovery Processes, *Elementary School Mathematics, Instruction, Instructional Materials, Mathematics, Objectives, *Secondary School Mathematics

Identifiers-The Madison Project

This pamphlet gives a brief introduction into the objectives, assumptions, content, activities, and materials of the Madison Project. Reported are the objectives of the program for the student and for the teacher. The mathematical content is listed under 22 topics. It is indicated that the availability of the materials may vary extensively from time to time due to extensive development activities. Criteria for the selection of the topics are listed also. Pertinent references that relate to objectives, content, and instructional procedures are presented. Materials produced by the project and services and activities of the project are listed. (RP)

ED028948

BR-5-1172
PA24



THE MADISON PROJECT

SYRACUSE UNIVERSITY
WEBSTER COLLEGE

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
OFFICE OF EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL OFFICE OF EDUCATION POSITION OR POLICY.

OEC-6-10-183

THE MADISON PROJECT—

A BRIEF

INTRODUCTION

TO MATERIALS

AND ACTIVITIES

ROBERT B. DAVIS

SE 006 621

REVISED SEPTEMBER, 1965

First printing: October, 1962
Second printing: March, 1963
Third printing: June, 1963
Fourth printing: August, 1963
Fifth printing: May, 1964 (revised)
Sixth printing: October, 1964 (revised)
Seventh printing: September, 1965 (revised)

Financial support of various aspects of Madison Project work has been provided by the National Science Foundation, the Division of Educational Research of the U. S. Office of Education, the Alfred P. Sloan Foundation, the Fund for the Advancement of Education of the Ford Foundation, the Marcel Holzer Foundation, Educational Services Incorporated (Watertown, Massachusetts), and a group of industries and trade unions in the area of St. Louis, Missouri.

Madison Project offices are located at these addresses:

The Madison Project
Syracuse University
Syracuse, N.Y. 13210
315 GR 6-5571

The Madison Project
Webster College
St. Louis, Mo. 63119
314 WO 2-0440

The Madison Project
P.O. Box 1176
Weston, Conn. 06883
203-227-2095

© Copyright 1965 by The Madison Project

"PERMISSION TO REPRODUCE THIS COPYRIGHTED MATERIAL HAS BEEN GRANTED BY Robert B. Davis, Director The Madison Project TO ERIC AND ORGANIZATIONS OPERATING UNDER AGREEMENTS WITH THE U.S. OFFICE OF EDUCATION. FURTHER REPRODUCTION OUTSIDE THE ERIC SYSTEM REQUIRES PERMISSION OF THE COPYRIGHT OWNER."

THE SCHOOL PROGRAM

The need to produce well-educated people is becoming more and more the central problem of our society, and within education, mathematics and science are assuming ever greater importance. Unless the basic nature of our society were suddenly -- and unexpectedly -- to change, there is no alteration of this process anywhere in sight.

This situation has given rise to the various "new mathematics" and "new science" projects of which the Syracuse University-Webster College Madison Project is one.

Because of the resources now being focused on scientific and mathematical education -- and even more because of the great need -- it can no longer be assumed that arithmetic will be taught in grades 1 - 8, algebra in grades 8, 9, and 11, geometry in grade 10, and "advanced algebra" or trigonometry in grades 11 and 12. Nor can the development of a high-quality "modern" mathematics program any longer be regarded as mainly a matter of "adopting a new textbook series." Rescheduling school classes, reconsideration of educational philosophy, considerable further teacher education, the introduction of laboratory experiences in mathematics, and less reliance on textbooks all tend to play a role in building a "modern" mathematics program.[1]

Similar fluidity exists in the science courses.

This is a very provocative situation. For those who prepare materials

---

[1] As evidence of what this can mean in actual practice, we might cite the highly sophisticated mathematics program at Nova High School, in Florida, developed by Burt Kaufman, and various quite innovative programs developed in England by Leonard Sealey, James Denny, and various members of the Association of Teachers of Mathematics. These programs are radically different from anything traditionally used in either the United States or England.

to be used in the schools, and for those in the schools who must plan the educational program, the comfortable guideposts are no longer trustworthy.

What can be put in their place?

The answer seems to be that, for the immediate future, considerable variety, uncertainty, experimentation, and reappraisal will be the order of the day.

A BRIEF STATEMENT ON OBJECTIVES

The broadest objective of the Madison Project is to use mathematics as an approach to the task of improving the quality of pre-college education, particularly in relation to giving students a deeper sense of involvement in the process of their own education, and to increasing the sense of vitality and relevance of educational experiences.

The more specific objectives of the Madison Project might be listed briefly under several headings: objectives for the student, objectives for the teacher, and objectives for the curriculum.

Objectives for the Curriculum. The Project has three kinds of objectives that relate to the evolution of the school mathematics curriculum. First, especially for grades 2 through 8, the Project seeks to broaden the curriculum. The traditional curriculum for these grades was concerned almost exclusively with the algorithms of arithmetic, with fractions, ratio, per cent, and applications to retail sales situations. This is, today especially, too narrow a slice of the world of mathematics. The Project seeks to broaden this curriculum by introducing, in addition to arithmetic, some of the fundamental concepts of algebra (such as variable, function, the arithmetic of signed numbers, open sentences, axiom, theorem, and derivations), some fundamental concepts of coordinate geometry (such as graph of a function), some ideas of logic (such as implication), and some work on the relations of mathematics to physical science.

Why is this broadening thought necessary? For one reason, because the

exclusive concentration on the algorithms of arithmetic is not representative of the mathematics that today's child needs to learn. There are, however, other reasons: arithmetic cannot be clearly understood all by itself -- it becomes clearer as one sees it in relation to algebra and coordinate geometry. Moreover, the opportunity for children to use arithmetic in creative, original, and exciting ways is not very great until one combines arithmetic with algebra, geometry, and science -- and then it becomes very great indeed. (This is perhaps somewhat analogous to saying that batting a ball would not be very gratifying if one had no experience with, or knowledge of, the game of baseball.)

Second, the Project attempts to instill a more creative flavor into the school mathematics curriculum. To attempt to describe this in any detail would be a lengthy undertaking, but one example may make it somewhat clearer.

In an actual occurrence in Weston, Connecticut, a third grade teacher was discussing the subtraction problem

$$\begin{array}{r} 64 \\ -28 \\ \hline \end{array}$$

and was saying something of the familiar sort "I can't take 8 from 4, so I regroup the 64 as 50 plus 14" (or whatever), when a third grade boy named Kye interrupted:

"Oh, yes, you can. Four minus eight is negative four ...

$$\begin{array}{r} 64 \\ -28 \\ \hline -4 \end{array}$$

and twenty from sixty is forty

$$\begin{array}{r} 64 \\ -28 \\ \hline -4 \\ 40 \end{array}$$

and forty and negative four is thirty-six

$$\begin{array}{r} 64 \\ -28 \\ \hline -4 \\ 40 \\ \hline 36 \end{array}$$."

This is a remarkable instance of original creative work by a student -- this delightful algorithm was made up by a third grade boy! -- and it afforded considerable pleasure to the class and to the teacher.

Now, there are many aspects of this episode that are of interest; let's observe two: in the first place, this invention could not have occurred if the children had not learned the arithmetic of signed numbers previously, in grade two. Secondly, Kye's remark would probably have been a source of consternation in a traditional setting, but, in the present setting, the teacher listened to Kye, tried to understand and appreciate his contribution, and the result was excitement and pleasure at the recognition of an original (and valuable) idea!

This last aspect is an important part of what we mean when we speak of developing "a more creative flavor" to the mathematical experiences that the child meets in school. In large part it means listening to the children, and recognizing that children are not merely able to discover and invent mathematics for themselves, but they have been attempting to do it for many years, and have ordinarily been rebuffed by most versions of "traditional" teaching.

The third objective for the curriculum is to achieve greater variety in the child's experiences with mathematics, and more active student participation. Not merely reading, writing, and reciting, but also participating in lively seminar discussions, working with physical apparatus, playing games, and making up some of your own mathematical systems or algorithms, etc.

Objectives for the Student. The Madison Project seeks to develop, and help schools to implement, a mathematics program that will help the student to:

i. develop his ability to discover patterns in abstract situations;

ii. develop a habitual use of "exploratory behavior" that goes beyond anything the teacher calls for explicitly, and investigates "what would happen if ...";

iii. acquire a set of mental symbols that will let him think creatively about mathematical situations;

iv. learn the really basic ideas of mathematics, such as variable, function, graph, matrix, isomorphism, implication, and so on;

v. acquire a reasonable mastery of important techniques;

vi. know basic mathematical facts -- such as, for example, the fact that $^{-}1 \times {}^{-}1 = {}^{+}1$.

The objectives listed above might be called "cognitive" or "intellectual" matters. They become fully effective only when they are accompanied by the following sort of "non-cognitive," "value," or "attitude" attributes, the development of which we would also list as objectives:

vii. a belief that mathematics is discoverable (Gauss didn't learn all he knew from his teachers and his textbooks -- and neither did the students in Johnny's own 5th grade class!);

viii. a realistic assessment of one's own ability to discover mathematics;

ix. a recognition that mathematics is incomplete and open-ended; there are unexplored frontiers on every side;

x. an honest self-critical ability;

xi. an appreciation of -- even a commitment to -- the value of abstract rational analysis, in its proper place;

xii. an appreciation of the value of "educated intuition" and shrewd speculations;

xiii. a feeling that mathematics is fun and worthwhile;

xiv. an appreciation for the history of the development of human culture (in which mathematics has, in fact, played a surprisingly large role);

xv. an appreciation of pure mathematics for its own sake, together with an appreciation of the scientific uses of mathematics.

Finally, the Project has objectives for the teacher. It is not enough to expect the child to grow from a fourth grader in 1965 to a fifth grader in 1966, etc. Even if we provide -- as we usually fail to do -- for the curriculum to grow from the curriculum of 1965 to the curriculum of 1966, that is still not enough. We adults must also grow from our 1965 selves to our new 1966 selves, and so on. This applies to teachers as much as to any other adult.

In particular, he who would teach mathematics to children must live in the world of the child, he must live in the world of mathematics, and he must live in the world of modern adult society. Claiming citizenship in these three worlds -- or in any one of them -- is not an easy matter, and no one's claim is totally valid. It is an objective of Madison Project materials and services to try to advance the legitimacy of the teacher's claim to citizenship in all three worlds: to gain an ever deeper understanding of children, and of mathematics, and of the human condition in the twentieth century.

References Concerning Objectives

Davis, Robert B. "The Evolution of School Mathematics," Journal of Research in Science Teaching. Vol. 1, 1963, pp. 260-264.

----------. "Goals for School Mathematics: The Madison Project View," Journal of Research in Science Teaching. Vol. 2, Issue 4, December, 1964, pp. 309-315.

Davis, Robert B. (Continued)

----------. Discovery in Mathematics: A Text for Teachers. Addison-Wesley Publishing Co., Inc. Reading, Massachusetts, 1964.

----------. "The Madison Project's Approach to a Theory of Instruction," Journal of Research in Science Teaching, Vol. 2 (1964), pp. 146 - 162.

----------. "Some Remarks on 'Learning by Discovery'." Available from the Madison Project (1965).

----------. "Mathematics" (Chapter 6 in the volume New Curriculum Developments, edited by Glenys G. Unruh). Association for Supervision and Curriculum Development, 1201 Sixteenth St. N.W., Washington, D.C. 20036 (1965), pp. 46 - 56.

----------. Matrices, Logic, and other Topics: A Text for Teachers. Addison-Wesley Publishing Co. (in preparation).

Goals for School Mathematics. The Report of the Cambridge Conference on School Mathematics, Houghton Mifflin Co., Boston, Massachusetts, 1963.

Mearns, Hughes. Creative Power. Dover Publications, New York, 1958.

Polya, George. Mathematical Discovery. On Understanding, Learning and Teaching Problem Solving. Vol. 1, Wiley, 1965.

Schwab, Joseph J. "The Teaching of Science as Enquiry," in J. J. Schwab and Paul Brandwein, The Teaching of Science, Harvard University Press (1964), pp. 1 - 103.

EDUCATIONAL HYPOTHESES

The Madison Project materials and activities incorporate certain hypotheses or operating assumptions. One or two of these need to be mentioned here.

Experience. For younger children (grades K - 8, say) the Project assumes that, in general, considerable experience with basic concepts and techniques should precede any formal instruction. This "experience" stage may be of several years duration and utilizes relatively flexible instructional methods.

To give some examples: in "traditional" programs, children often learn to divide fractions by the "invert and multiply" rule long before they have any real notion of what a fraction is, or what the division of fractions means. The traditional program did provide excellent informal experiences with whole numbers in kindergarten and first grade without any formal instruction. Children counted milk bottles, took attendance, kept track of the days of the month, kept track of "how many days until Christmas," played games that involved whole numbers, and so on. Similar experiences were not provided in the case of fractions. On the contrary, formal instruction concerning fractions was built upon a quite inadequate experiential background. As a result,

$$\frac{3}{4} \div \frac{1}{2}$$

had no meaning for most children even though they were asked to memorize the "invert and multiply" rule. Madison Project materials seek to introduce several years of informal experience with fractions before arriving at the "invert and multiply" stage of formality.

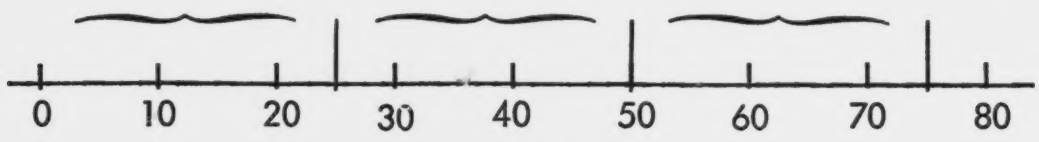
If [1]

$$\frac{75}{25}$$

---

[1] We are here using the heavy horizontal bar to mean "divided by"; thus, $\frac{1}{\frac{1}{2}}$ means the same thing as $1 \div \frac{1}{2}$.

means "how many times does 25 go into 75," which you can visualize on a number line as

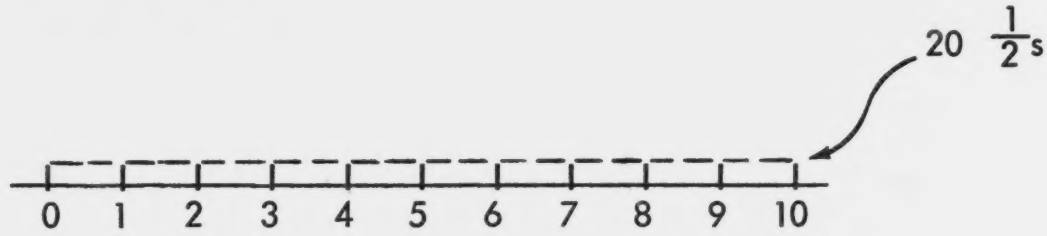


,

then

$$\frac{10}{\frac{1}{2}}$$

should mean "how many times does $\frac{1}{2}$ go into 10," which you can also visualize on a number line as



Consequently, <u>without formal instruction</u>, it is clear to children that

$$\frac{10}{\frac{1}{2}} = 20$$

$$\frac{10}{\frac{1}{3}} = 30$$

$$\frac{10}{\frac{1}{7}} = 70.$$

and so on.

Equally,

$$\frac{\frac{1}{2}}{\frac{1}{4}}$$

should mean "how many times does $\frac{1}{4}$ go into $\frac{1}{2}$," and a mental visualization of the number line makes it clear that

$$\frac{\frac{1}{2}}{\frac{1}{4}} = 2$$

Given enough experience of this kind, children who have become accustomed to looking for and discovering patterns will come up with the "invert and multiply" rule without the need for much formal instruction at all.[1]

Again, Madison Project materials provide long informal experience with equations before getting into formal consideration of equations. They provide as much as several years of informal experience with implication (for example, through the medium of games involving implication) before formal consideration of implication. They provide only informal experiences with matrices before any formal consideration of matrices. Similarly, for nearly every basic concept and technique that is considered.

---

[1] In fact, Madison Project materials for fractions cover three stages of the child's growth: first, informal relatively unstructured exploration; second, discovery of patterns and generalizations; third, proof (from axioms) of the relevant theorems. The "traditional" program in the recent past (in the United States) offered none of these three stages, but consisted of the single stage of being told rules and practising these rules.

This initial stage of flexible, informal instruction is conducted as much as possible on an exploratory, "fun" basis without rigid specifications of required attainment levels, etc.[1]

"Discovery." As mentioned earlier, the Project assumes that the ability to discover patterns in abstract material is one of the most essential mathematical skills -- quite possibly the most essential skill. Consequently, a main goal of Project teaching is to give children as much experience as possible in discovering patterns.

Unfortunately, the "discovery" approach to learning means quite different things to different people. It is strongly suggested that no one form an opinion of what the Madison Project means by "discovery" teaching until after viewing some Project films (which show actual classroom lessons), or visiting Project classes at one of the experimental schools.

The various "new mathematics" projects do not all mean the same kind of teaching when they speak of "discovery." The difference may well be quite important.

Unity of "Method" and "Content." For various historical reasons there has developed considerable separation between what is generally thought of as "teaching method" and what is thought of as "mathematical content."

---

[1] It should be clear from this discussion that "experience" is not being used to mean "concrete experience." The vast majority of the "experiences" which the Project has developed for children deal with abstract matters. This sometimes seems incongruous to adult observers, but it does not seem so to the children.

However, since 1964, the Project has been working to include a larger number of concrete experiences, such as studying the velocity and acceleration of an actual automobile, etc. This new emphasis reflects, in part, the influence of Caleb Gattegno, Leonard Sealey, and Lauren Woodby on the Project's operational orientation.

Recent developments in mathematics teaching have rendered this separation untenable (if indeed, it ever was at all viable). It is not very descriptive of a school program to say that it "includes the topic of simultaneous equations," or that it "is based upon the use of sets," or that it "includes a careful treatment of the real numbers." The effectiveness of a program must be judged not by "what is taught," but rather by what is learned. This includes the attitudes that are learned and the degree of facility, originality, resourcefulness, etc., that is developed in the students.

The Madison Project regards "content" and "method" as equally important and inseparable. It might be pointed out that the most effective mathematics teachers on every level have always held this view, although explicit discussion of such matters may have been abhorrent to many of them.

Because of this unity of what is taught and how it is taught, the Project relies heavily upon a series of 16mm. sound motion picture films showing actual classroom lessons. As already suggested, the best introduction to Project activities and materials is visiting one of the Project's experimental schools, and the next-best method is viewing some of the Project's films.

MATHEMATICAL CONTENT

The Madison Project materials provide a supplementary program -- not a substitute for the standard program. The mathematical content of Project materials centers around these topics:

1) Practice in counting discrete objects, such as pebbles, etc.

2) The concepts of addition, multiplication, subtraction, and division.

3) Place-value numerals, and algorithms for adding, subtracting, multiplying, and dividing.

4) Experience with fractions.

5) Experience with angles, length, area, and volume.

6) Coordinate geometry.

7) Experience with implication and contradiction.

8) An axiomatic approach to algebra.

9) Experience with functions.

10) Experience with matrices.

11) Experience with similar triangles.

12) The trigonometric functions.

13) Vectors and forces; displacement, velocity, and acceleration.

14) Extending number systems.

15) Experience programming a digital computer.

16) Average, variance, inner-quartile range, problems of measurement.

17) Truth tables and inference schemes; many-valued logics, and two-valued logic.

18) Rate of change; graphical differentiation and graphical integration.

19) Limit of a sequence; convergence.

20) Finite difference methods.

21) Mathematical induction.

22) A comparison of three approaches to geometry -- Euclidean, Cartesian, and vector.

Not all of the topics mentioned above are at the same state of completion and availability. Since the availability of appropriate materials, developed either by the Project or by others, changes rapidly, it is best to seek up-to-date lists from the various publishers, authors, and experimental projects that are active in this area.

SELECTION OF TOPICS

The nature of Project materials is somewhat revealed by the criteria that are used in including or excluding topics. First, a list of basic concepts and techniques is prepared. This list includes:

The concept of variable

The concept of open sentence and truth set

Classification of statements as "true" or "false"

Locating points on Cartesian coordinates

The arithmetic of signed numbers

The concept of function

The graph of a function, and graphical representation of a truth set

Systems of simultaneous open sentences

Implication

Contradiction

Axioms, theorems, and derivations (mainly in algebra)

Identities

"General form" of equations; the use of variables in writing "any" quadratic equation, "any" 2 x 2 matrix, etc.

Matrices

Vectors

Extensions of number systems

... and other topics of a similarly fundamental nature.

Specific classroom experiences are now designed and included in the Project materials if they satisfy five criteria:

1) They are directly related to the fundamental concepts and techniques on the list above.

2) They provide an active role for the children. Passive experiences such as listening to a lecture or reading an extended exposition are generally avoided.

3) They provide considerable opportunities for "discovery" by the children.

4) The mathematical concepts are learned in an appropriate context, rather than presented in isolation out of context.

5) The experiences seem to be appropriate to children at the age in question.

For example, one of the first topics studied is the "discovery" of the coefficient rules for a quadratic equation written in a standard form. It is not the purpose of this lesson to teach quadratic equations. Rather, it is intended to provide experience with variables, substituting into equations, the truth set of an open sentence, and some simple arithmetic of signed numbers. In line with the five criteria above, we have found this use of quadratic equations to be admirably suited to this task. Routine "drill" exercises on the arithmetic of signed numbers would not be suitable and are not used.

AN "INFORMAL" "FORMAL" APPROACH

Underlying Project materials are two seemingly contradictory hypotheses.

On the one hand it is assumed that children need much exploratory or preliminary experience with mathematical situations, that generalization from

instances has a large role to play, and even that physical experiments and pseudo-geometric representations of mathematical situations are important and must be reasonably prominent. This might be called a "heuristic" or "informal" approach to learning mathematics. This emphasis is further accentuated by the very marked informality which characterizes classroom lessons of Project materials, as can be seen in the films. (Project classes, for example, are usually "noisy." The children are enthusiastic and <u>active</u>.)

Seemingly opposing this is the Project's insistence upon a formal axiomatic foundation as the "legal" or "official" basis for arithmetic and algebra. For example, the identity

$$\square + \square = 2 \times \square$$

is regarded as a theorem implied by a stated set of axioms. Constructing original proofs of such theorems is a very prominent student activity. As another example, for $a \neq 0$, $b \neq 0$, the identity

$$\frac{a}{b} \times \frac{b}{a} = 1$$

is a <u>theorem</u> to be proved from the axioms.

This "formal" emphasis is accentuated by the Project's insistence upon precise, explicit notations with a minimum of "conventions" (we do not usually omit parentheses or multiplication signs, for example, and <u>additive inverses</u> we carefully and consistently distinguish from <u>negative numbers</u>.)

This apparent contradiction of "formal" and "informal" approach is not an actual contradiction. In actual experiments with children, "formal" proofs are just as "exciting," just as much "fun" to make as are physical experiments or heuristic searches for "answers."

What in fact we are seeking is <u>clean</u> mathematics, <u>honest</u> mathematics, <u>lively</u> mathematics, and <u>exciting</u> mathematics. Far from being antithetical, these are very harmonious ingredients that, in the long run, are more easily realized in <u>combination</u> than they could be in isolation from one another.

USE OF "□" AND "△"

Project materials use several "modern" notations mainly adopted from the pioneering work of Beberman and the University of Illinois U.I.C.S.M. These include the use of distinctive symbols for positive and negative numbers ($^{+}3$, $^{-}7$) that avoid confusion with operational symbols for addition and subtraction.

$$^{+}3 - {}^{+}5 = {}^{-}2$$

$$^{-}2 + {}^{-}8 = {}^{-}10 ,$$

and a distinctive symbol to indicate the "opposite" or "additive inverse" of a number:

$$^{\circ}\square$$

$$^{\circ}(A + B) = {}^{\circ}A + {}^{\circ}B$$

$$^{\circ}(^{-}7) = {}^{+}7$$

$$^{\circ}(^{+}1) = {}^{-}1$$

$$^{\circ}(0) = 0$$

$$\square + {}^{\circ}\square = 0$$

and so on.

However, much the most distinctive and controversial of the "modern" symbols is the use of □, △, ▽, etc. to denote <u>variables</u>. Some explanation of these symbols should perhaps be given.

1) For whatever reason, "□" is more easily used with children than "x" is. For example,

$$3 + \square = 5$$

invariably suggests to 4th or 5th graders that they could write "2" in the "□":

$$3 + \boxed{2} = 5$$

This is particularly important when the method of teaching seeks to avoid the use of exposition.

2) David Page has pointed out that "□" asks the right question; namely, what can I put there? Letters do not do this as naturally.

3) Page further points out that "□," "△" notation has the great advantage of showing a general pattern at the same time that it exhibits a special case.

General pattern: $\square \times (\triangle + \triangledown) = (\square \times \triangle) + (\square \times \triangledown)$

Special Case: $2 \times \left(4 + \frac{1}{2}\right) = \left(2 \times 4\right) + \left(2 \times \frac{1}{2}\right)$

Both Exhibited Simultaneously: $\boxed{2} \times \left(\triangle\!\!\!

the recommended usage in ordinary reading and writing of English. (Indeed, mathematical use is even more complex than simple permutations, for I can write

c a t

and understand it to mean "eel"; "e" can be a replacement for the variable "c," and also for the variable "a.")

Thus, the child is pulled in two quite different directions, a discomfort that can be avoided by the use of "□" and "△" to denote variables.

SOME RELEVANT REFERENCES ON PEDAGOGICAL MATTERS

Ashton-Warner, Sylvia. Teacher. Simon and Schuster, New York, 1963.

Avers, Paul W. "A Unit in High School Geometry Without the Textbook," The Mathematics Teacher, Vol. LVII, No. 3 (March, 1964), pp. 139 - 142.

Boole, Mary Everest. The Preparation of the Child for Science. Oxford, 1904.

Boulle, Pierre. The Test. Popular Library, New York, 1960.

Brecher, Ruth and Edward Brecher. "Gifted Children Need Freedom to Learn," Parents' Magazine (June, 1962), p. 44 ff.

Bruner, Jerome S. "The Cause of Cognitive Growth," American Psychologist, Vol. 19, No. 1 (1964), pp. 1 - 15.

----------. The Process of Education. Harvard University Press, 1963.

Cantor, Nathaniel. The Dynamics of Learning. Henry Stewart Inc., New York, 1961.

Clarkson, David. "Taxicab Geometry, Rabbits, and Pascal's Triangle-Discoveries in a Sixth-Grade Classroom," The Arithmetic Teacher, Vol. IX, No. 6 (October, 1962), pp. 308 - 313.

Courtis, Stuart A. "Our Choice: Revolution or Destruction," Educational Leadership, Vol. 20, No. 8 (May, 1963), pp. 520 - 522.

Davis, Robert B. "Emotion and Thought," The Mathematics Teacher, Vol. XLVIII, No. 3 (March, 1955), pp. 133 - 142 (reprinted in M. Vere Devault's Improving Mathematics Programs, Merrill Books, Columbus, Ohio, 1961, pp. 108 - 121).

Day, Robert and Robert Hamblin. "Some Effects of Close and Punitive Styles of Supervision," American Journal of Sociology, Vol. LXIX, No. 5 (March, 1964), pp. 499 - 510.

de Charms, Richard, Virginia Carpenter, and Aharon Kuperman. "The 'Origin-Pawn' Variable in Person Perception," Sociometry (to appear).

Dienes, Z.P. The Power of Mathematics. Hutchinson Educational Limited, London, 1964, especially pp. 17 - 25.

Goodman, Paul. Compulsory Mis-education. Horizon Press, New York, 1964, pp. 53, 54, and p. 81.

Hawkins, David. "On Living in Trees," Karl Muenzinger Memorial Lecture, University of Colorado, 1964.

Henry, Jules. "American Schoolrooms: Learning the Nightmare," Columbia University Forum. Vol. 6, No. 2 (Spring, 1963), pp. 24 - 30.

Holt, John. How Children Fail. Pitman, 1964.

Huxley, Aldous. "The Education of an Amphibian," contained in Tomorrow and Tomorrow and Tomorrow, Signet Books, 1964.

Kaufman, Bel. Up the Down Staircase. Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1964.

Kelley, Earl C. The Workshop Way of Learning. Harper, New York, 1951.

Kersh, Bert Y. "Learning By Discovery: What Is Learned?" The Arithmetic Teacher, Vol. 11, No. 4 (April, 1964), pp. 226 - 232.

Neill, A.S. Summerhill. Hart Publishing Co., New York, 1960.

Page, David A. "Well-Adjusted Trapezoids," Updating Mathematics, Croft Educational Services, 100 Garfield Ave., New London, Connecticut, Vol. 4, No. 8, Sec. 11 (1962).

Pincus, Morris. "An Adventure in Discovery," The Arithmetic Teacher, Vol. 11, No. 1 (January, 1964) pp. 28 - 29.

Reik, Theodor. Listening With the Third Ear. Farrar, New York, 1948.

Rockcastle, Verne N., et al. Piaget Re-discovered. A report of the Conference on Cognitive Studies and Curriculum Development, March, 1964, School of Education, Cornell University (edited by Verne N. Rockcastle and Richard E. Ripple).

Rogers, Carl R. Client-Centered Therapy. Houghton Mifflin Co., Boston, 1959.

Sawyer, W. Warwick. Vision in Elementary Mathematics. Penguin Books, Baltimore, Maryland, 1964.

Smedslund, Jan. "Internal Necessity and Contradiction in Children's Thinking," Journal of Research in Science Teaching, Vol. 2, No. 3 (September, 1964), pp. 220 - 221.

Synder, Henry D. "An Impromptu Discovery Lesson in Algebra," The Mathematics Teacher (October, 1964), pp. 415 - 416.

Snygg, Donald. "A Learning Theory for Curricular Change," Chapter 11 of Using Current Curriculum Development. Association for Supervision and Curriculum Development, NEA, 1201 Sixteenth St., N.W., Washington 36 D.C. (1963), pp. 109 - 115.

Tolman, Edward Chace. "Cognitive Maps in Rats and Men," Behavior and Psychological Man, University of Chicago Press (1958), Chapter 19.

Torrance, E. Paul. "Creativity," Department of Classroom Teachers, American Educational Research Association of the National Education Association. Research Pamphlet Series, What Research Says to the Teacher, No. 28 (April, 1963).

Whitehead, Alfred North. Aims of Education. Mentor Books, Macmillan Co., New York, 1929.

AVAILABLE MATERIALS AND SERVICES

Although the activities and services of the Madison Project are probably more important than any of the specific materials which the Project has produced, we might best begin with a listing of some relevant materials, produced by the Project, or by others, for use in teacher education or else directly in the classroom:

Materials for Teacher Education

In-Service Course I. This "packaged" course combines films and printed materials, and is intended for an in-service course meeting for at least 12 sessions, and possibly (depending upon timing and pace) for as many as 24. (For further description, see Newsletter #1.)[1]

In-Service Course II. A sequel to Course I. (Cf. Newsletter #1.)

Davis, Robert B. Discovery in Mathematics: A Text for Teachers. Addison-Wesley Publishing Co., Inc., Reading, Massachusetts, 1964.

----------. Experimental Course Report/Grade Nine. (Report #1, June, 1964). Available from the Madison Project.

----------. Matrices, Logic, and other Topics: A Text for Teachers. Addison-Wesley Publishing Co. (In preparation.)

Films showing actual classroom lessons. (Cf. Newsletter #1.)

---

[1] Newsletter #1 (July, 1965), available from the Madison Project.

Fletcher, T.J., D.H. Wheeler, T.H.F. Brissenden, W.B. Brookes, D.G. Tahta, et al. Some Lessons in Mathematics. A Handbook on the Teaching of "Modern" Mathematics. Cambridge University Press, 1964.

Machtinger, Doris Diamant. Experimental Course Report/Kindergarten (Report #2, July, 1965), available from the Madison Project.

Page, David and UICSM Staff. "Arithmetic with Frames," available from the University of Illinois Arithmetic Project, 1207 West Stoughton Street, Urbana, Illinois (1960).

----------. "Do Something About Estimation!" Updating Mathematics, Croft Educational Services, 100 Garfield Avenue, New London, Connecticut, Vol. 2, No. 8 (April, 1960).

----------. "Well-Adjusted Trapezoids," Updating Mathematics, Croft Educational Services, Vol. 4, No. 8, Sec. 11 (1962).

Sawyer, W. Warwick. Visions in Elementary Mathematics. Penguin Books, Baltimore, Maryland, 1964.

Materials for Use in Class

Cohen, Donald. "A Lesson on Absolute Value," The Arithmetic Teacher, Vol. 11, No. 8 (December, 1964), pp. 561 - 562.

Davis, Robert B. Discovery in Mathematics: Student Discussion Guide. Addison-Wesley Publishing Co., Inc., Reading, Massachusetts, 1964.

----------. Matrices, Logic, and other Topics: Student Discussion Guide. Addison-Wesley Publishing Co. (in preparation).

Faber, Norman, Phyllis Gross, and Robert Hansen. Mathematics and Living Things. School Mathematics Study Group (SMSG), Stanford University, 1964.

Merrick, Paul. Elementary Science Study (ESS) of Educational Services Incorporated (ESI), Gases and Airs, Houghton Mifflin Co., Boston, 1965.

----------. The Shell Game -- An Experiment in Estimation and Measurement. The Webster Institute of Mathematics, Science, and Arts (WIMSA), Webster College, Webster Groves, Missouri, 1965.

Page, David A. Number Lines, Functions, and Fundamental Topics. Macmillan, 1964.

"Cuisenaire rods" and other materials are available from the Cuisenaire Company of America, Inc., 9 Elm Ave., Mt. Vernon, New York.

Dr. Z. P. Dienes, of the University of Adelaide, has many interesting physical materials available for use in class, such as his famous MAB blocks for place-value numerals, etc.

Elementary School Science Project (ESSP) materials, University of California, Berkeley, California.

The film Straight-Line Kinematics, Physical Science Study Committee (PSSC), Educational Services Incorporated, 164 Main Street, Watertown, Massachusetts.

SERVICES AND ACTIVITIES

The Madison Project does not produce primarily materials. Its main emphasis is in providing various services and pursuing various activities. Some of the most important among these are described below.

1. Work with "Lighthouse Schools." Since 1958, the Madison Project has worked closely with various school systems that have specialized in providing the highest possible quality in education -- schools, or school systems, such as: Weston, Connecticut; Scarsdale, New York; Greece, New York; West Irondequoit, New York; Lincoln, Massachusetts; Ladue, Missouri; Upwood Primary School, Huntingdon, England; and others. (Recent additions to this list include Redding, Connecticut; St. Thomas Choir School, in New York City; the Webster College Experimental School; and Nova High School, in Fort Lauderdale, Florida.)

This collaboration has been a two-way street. The Project has attempted to bring its scholarship and experience to bear on the task of curriculum development in these schools, and it has learned from these schools what it means to try to develop a very high-quality sequence of mathematical learning experiences for pre-college children.[1]

The task of sharing the fruits of this collaboration with the world at

---

[1] Obviously, many teachers and administrators have contributed to this activity. Among those especially prominent have been: Jane Downing, Beryl Cochran, Doris McLennan, William Bowin, J. Robert Cleary, Harold Howe, Inge Clark, Dante Zacavish, Gordon Clem, Burt Kaufman, Ruth Hertlein, Gerald Baughman, Frank Duval, Dr. Frank Morley, James Denny, Elizabeth Bjork, Marianne Ockerbloom, Elizabeth Herbert, Lyn McLane, Augustus Young, Gilbert Brown, Herbert Barrett, Thomas Davis, Marie Lutz, David Robinson, Frank van Atta, Adelyn Muller, and Sister Francine, S.L. (This is only a partial list; the work of the Project has been carried on by many others, as well.)

large is challenging and arduous. One attempt has been made via 16 mm. motion picture films which show actual classroom lessons in some of these schools. In addition, various written reports are available, especially:

Robert B. Davis, (i) Experimental Course Report/Grade Nine (Report #1, June, 1964), available from the Madison Project.

(ii) A Modern Mathematics Program As It Pertains to the Interrelationship of Mathematical Content, Teaching Methods, and Classroom Atmosphere. (The Madison Project), 1963. Report for 1963 submitted to the Commissioner of Education, U.S. Department of Health, Education and Welfare.

(iii) "Report on Madison Project Activities, September, 1962 - November, 1963," report submitted to the National Science Foundation in December, 1963.

J. Robert Cleary, "A Study of Test Performance in Two Madison Project Schools and One Control School," available from the Madison Project.

and also an audio-tape recording:

Tape-recording No. D-1, available from the Madison Project.

2. Work with Large Cities. Under the leadership of Dr. Samuel Shepard of St. Louis, Dr. Evelyn Carlson, Bernice Antoine, and Dr. Jerome Sachs of Chicago, Dr. John Huffman of San Diego County, Emma Lewis and Gail Saliterman of Washington, D.C., and others, the Madison Project has, since 1963, conducted a collaboration with the school systems of some of our largest cities (St. Louis, Chicago, San Diego, and Washington, D.C.) which builds upon, and in many respects resembles, the work with the "light-house" suburban and private schools. Of course the problems of our greatest urban centers are different, including cultural deprivation, mobility both of teachers and of students, financial limitations, and -- perhaps above all --

tremendous size. Nonetheless, the Project's "urban program" has given every evidence of significant progress, and is now being reported via films showing actual classroom lessons, and via written reports.

Films: Guessing Functions

Postman Stories

Graphs and Truth Sets

3. Teacher Education. Obviously, large-scale teacher education is part of the "urban" program described above. The Project is, however, involved in many other forms of teacher education, especially the undergraduate college "pre-service" education at Webster College, the graduate in-service MAT program at Webster College, the NSF Summer Institute program at Syracuse University, and a special student-teaching program operated by the Valley View schools, Overland Park, Kansas, under the leadership of Adelyn Muller.

PARTIAL LISTING OF PROJECT PERSONNEL

Director: Robert B. Davis, Professor of Mathematics and Professor of Education, Syracuse University; Visiting Professor of Mathematics, Webster College

Co-ordinator for Syracuse University: Donald E. Kibbey, Chairman, Mathematics Department, Syracuse University

Co-ordinators for Webster College:

Sister M. Jacqueline, S.L., President, Webster College

Katharine Kharas, Chairman, Mathematics Department, Webster College

Co-ordinator for Weston, Connecticut Experimental Center: Beryl S. Cochran

Administrative Assistant: Martha Bowen

In charge of manuscript production: Bernice Talamante

In charge of film distribution: Louise Daffron

Demonstration Teachers:

Donald Cohen
Louis Cohen
Doris Machtinger
Knowles Dougherty
Donna Doyle
Joan O'Connell
Gerald Glynn

Consultants, Advisors, and Occasional Advisors:

Professor Richard de Charms, Washington University, psychology (motivation)

Dr. C. Brooks Fry, M.D., Los Angeles, psychiatry

J. Robert Cleary, Educational Testing Service, evaluation and description

Professor Carl Pitts, Webster College, psychology (motivation)

Leonard Sealey, Grey Friars, Leicester, Great Britain, mathematics education

Professor Jerome Bruner, Harvard University, psychology (cognition)

Professor Jerome Kagan, Harvard University, psychology (developmental)

Herbert Barrett, Weston, Connecticut, clinical psychology

Knowles Dougherty, Cambridge, Massachusetts, problems of cultural deprivation

Professor Earl Loman, M.I.T., physics

Professor Jerrold Zacharias, M.I.T., physics

Morton Schindel, President, Weston Woods Studios, film production

Dr. Malcolm Skolnik, M.I.T., physics

Professor William Walton, Webster College, physics

Professor Robert Karplus, University of California at Berkeley, physics

Professor Paul Merrick, Webster College, science teaching

Professor Emily Richard, Webster College, science teaching

Professor David Hawkins, University of Colorado, science and education

Dr. Gardner Quarton, M.D., Harvard Medical School, psychiatry

† Dr. Francis Friedman, M.I.T., physics and education

Professor Robert Exner, Syracuse University, mathematical logic

Professor Andrew Gleason, Harvard University, mathematics

Professor Frederic Mosteller, Harvard University, statistics

Professor Gail Young, Tulane University, mathematics

Professor Stewart Moredock, Sacramento State College, mathematics education

In addition, the Project has been heavily influenced by the remarks, writings, and example of Max Beberman, David Page, W. W. Sawyer, George Polya, Patrick Suppes, Dr. C. Harry Bagenstose, M.D., Thomas Hastings, Paul Rosenbloom, Norman Levinson, Robert Rosenbaum, Morris Kline, E.E. Moise, Felix Browder, Douglas McGregor, Karl W. Deutsch, Kurt Lewin,† Cynthia Parsons, and Norbert Wiener.†

† Deceased